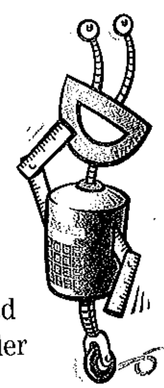
211 Properties

When you're working with real numbers, there are some things that are always true about how they behave. These things are called **properties**. Most of them are common-sense, but some of them aren't so obvious.

212 Commutative Properties

Sometimes order matters: You don't put on your socks after your shoes. But sometimes order doesn't matter: you *could* put on your shoes before your belt.

Commutative sounds like commute. That means "go back and forth." Addition and multiplication work either backward or forward.



213 Commutative Property of Addition

The **Commutative Property of Addition** is sometimes called the Order Property of Addition. It states that changing the order of addends does not change the sum. So, $a + b = b + a$.

How can knowing this property help you? One way is that you can use it to make it easier to add.

EXAMPLE 1: Add 26 + 18.

$26 + 18 = 44 \qquad 18 + 26 = 44$

So, $26 + 18 = 18 + 26$.

EXAMPLE 2: Add 25 + 147 + 75.

If you followed the order of operations rules, you'd add 25 + 147, then add 75. But you can use the Commutative Property to switch the order of 147 and 75.

MORE HELP
See 093

$$\begin{aligned} 25 + 147 + 75 &= 25 + 75 + 147 \\ &= 100 + 147 \\ &= 247 \end{aligned}$$

Adding 25 + 75 = 100, then 100 + 147 = 247, is pretty easy!

Commutative Property of Multiplication

The **Commutative Property of Multiplication** is also called the Order Property of Multiplication. It states that changing the order of factors does not change the product. So, $a \times b = b \times a$.

$5 \times 3 = 15$

$3 \times 5 = 15$

EXAMPLE: Multiply 5 × 167 × 2.

If you followed the order of operations rules, you'd multiply 5 × 167, then multiply by 2. But you can use the Commutative Property to switch the order of 167 and 2 to make it easier to multiply.

$$\begin{aligned} 5 \times 167 \times 2 &= 5 \times 2 \times 167 \\ &= 10 \times 167 \\ &= 1670 \end{aligned}$$

Multiply 5 × 2 = 10, then 10 × 167 = 1670.

215 **MATH ALERT Subtraction and Division Are Not Commutative**

Is $8 - 2$ equal to $2 - 8$? Is $8 \div 2$ equal to $2 \div 8$? No.

So, subtraction and division are *not* commutative.

CASE 1 BUT, you can rewrite a subtraction expression as addition and then use the Commutative Property of Addition.

EXAMPLE 1: Rewrite $8 - 2$ as a commutative expression.

$$8 - 2 = 6$$

Subtracting 2 is the same as adding $^-2$. $8 + {}^-2 = 6$

Addition is commutative. ${}^-2 + 8 = 6$

CASE 2 You can also rewrite a division expression as multiplication and then use the Commutative Property of Multiplication.

EXAMPLE 2: Rewrite $8 \div 2$ as a commutative expression.

$$8 \div 2 = 4$$

Dividing by 2 is the same as multiplying by $\frac{1}{2}$. $8 \times \frac{1}{2} = 4$

Multiplication is commutative. $\frac{1}{2} \times 8 = 4$

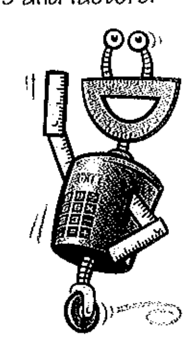
MORE HELP
See 188

216 Associative Properties

If you have a bunch of chores to do, you might save time by grouping them in a certain way. It makes sense to change the litter in the cat box, take out the trash, then wash up for dinner rather than to wash up *before* doing your other chores!

You associate with friends in groups. The associative properties are all about the ways you can group addends and factors.

217 Associative Property of Addition



The **Associative Property of Addition** states that changing the grouping of addends does not change the sum. So, $(a + b) + c = a + (b + c)$. Note that the order of the addends stays the same.

How can knowing this property help you? One way is that you can use it to make things easier to add.

EXAMPLE: Add 88 + 49 + 21.

MORE HELP
See 207–210

If you followed the order of operations rules, you'd add 88 + 49 and then add 21. But you can use the Associative Property to change the grouping so you can add 49 + 21 first.

$$\begin{aligned}(88 + 49) + 21 &= 88 + (49 + 21)\\ &= 88 + 70\\ &= 158\end{aligned}$$

Associative Property of Multiplication 218

The **Associative Property of Multiplication** states that changing the grouping of factors does not change the product. So, $(a \times b) \times c = a \times (b \times c)$.

You can use this property to make it easier to multiply.

EXAMPLE: Multiply 82 × 25 × 4.

If you followed the order of operations rules, you'd multiply 82 × 25, then multiply by 4. But you can use the Associative Property to change the grouping so you can multiply 25 × 4 first.

$$\begin{aligned}(82 \times 25) \times 4 &= 82 \times (25 \times 4)\\ &= 82 \times 100\\ &= 8200\end{aligned}$$

Distributive Property 219

When you distribute things, you spread them out. The Distributive Property lets you *spread out* numbers so they're easier to work with.

Distributive Property of Multiplication 220

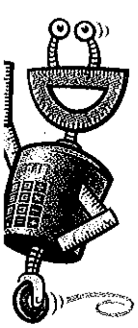
The **Distributive Property of Multiplication** states that for a, b, and c,

$a(b + c) = (ab) + (ac)$

$a(b - c) = (ab) - (ac)$

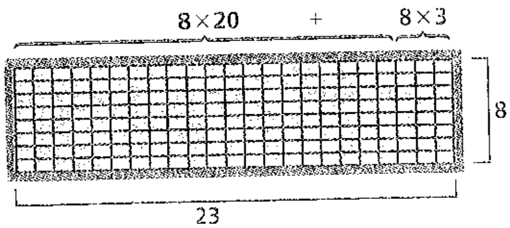
You can use this property to help you multiply in your head.

MORE ▶



EXAMPLE 1: Find the number of solar cells in an 8×23 array.

$$8 \times 23 = 8 \times (20 + 3)$$
$$= (8 \times 20) + (8 \times 3)$$
$$= 160 + 24$$
$$= 184$$



So, there are 184 solar cells in an 8×23 array.

EXAMPLE 2: Multiply 6×99.

$$6 \times 99 = 6 \times (100 - 1)$$
$$= (6 \times 100) - (6 \times 1)$$
$$= 600 - 6$$
$$= 594$$

99 is the same as 100 − 1. It's easier to multiply by 100 and 1 than it is to multiply by 99.

221 Making Division Distributive

MORE HELP See 188

The Distributive Property of Multiplication says that $a(b + c) = ab + ac$. Since dividing by a number is the same as multiplying by its reciprocal, you can say that $(b + c) \div a = \frac{1}{a}(b + c)$.

So, $(b + c) \div a = \frac{1}{a}(b + c) = \frac{b}{a} + \frac{c}{a}$.

EXAMPLE 1: Divide $(15 + 3) \div 5$.

$$(15 + 3) \div 5 = \tfrac{1}{5}(15 + 3)$$
$$= \tfrac{1}{5}(15) + \tfrac{1}{5}(3)$$
$$= 3 + \tfrac{3}{5}$$
$$= 3\tfrac{3}{5}$$

EXAMPLE 2: Divide $(15 - 3) \div 5$.

$$(15 - 3) \div 5 = \tfrac{1}{5}(15 - 3)$$
$$= \tfrac{1}{5}(15) - \tfrac{1}{5}(3)$$
$$= 3 - \tfrac{3}{5}$$
$$= 2\tfrac{2}{5}$$

222 Identity Elements

Identity elements are numbers that combine with other numbers without changing them. These properties are just common sense.

CASE 1 The Identity Element for Addition is 0 because $a + 0 = a$ and $0 + a = a$.

$$5 + 0 = 5$$
$$0 + 5 = 5$$

CASE 2 The Identity Element for Multiplication is 1 because $a \times 1 = a$ and $1 \times a = a$.

$$5 \times 1 = 5$$
$$1 \times 5 = 5$$

MATH ALERT No Identity Elements for Division and Subtraction

Is $a - 0$ always a? Yes.
Is $0 - a$ always a? Not unless $a = 0$.

Subtraction is not commutative. It doesn't work the same way when the numbers are reversed. So, you can't say that zero is an identity element for subtraction as it is for addition.

Is $a \div 1$ always a? Yes.
Is $1 \div a$ always a? Not unless $a = 1$.

Division is not commutative. It also doesn't work the same way when the numbers are reversed. So, you can't say that 1 is an identity element for division as it is for multiplication.

Inverse Elements

Inverse elements are numbers that combine with other numbers and result in identity elements—1 or zero.

MORE HELP
See 046–047, 187–192

CASE 1 Additive Inverse
A number added to its additive inverse always equals 0:

8 and ${}^-8$ are additive inverses.	$8 + {}^-8 = 0$
$\frac{3}{5}$ and ${}^-\frac{3}{5}$ are additive inverses.	$\frac{3}{5} + {}^-\frac{3}{5} = 0$
${}^-7.2$ and 7.2 are additive inverses.	${}^-7.2 + 7.2 = 0$

CASE 2 Multiplicative Inverse (usually called the **reciprocal**)
A number multiplied by its reciprocal always equals 1:

3 and $\frac{1}{3}$ are reciprocals.	$3 \times \frac{1}{3} = 1$
${}^-\frac{3}{8}$ and ${}^-\frac{8}{3}$ are reciprocals.	${}^-\frac{3}{8} \times {}^-\frac{8}{3} = 1$
4.6 and $\frac{1}{4.6}$ are reciprocals.	$4.6 \times \frac{1}{4.6} = 1$