Teaching Algebraic Linear Equations to Elementary School Students

by Henry Borenson, Ed.D.

It is thrilling to see the joyful expressions on the faces of third and fourth graders as they easily solve such equations as

$$2x + x + x + 2 = 2x + 10$$

and

$$2(x + 4) + x = x + 16.$$

Young students know that this kind of mathematics looks important. Hence, when they experience success with it, their pride and self-esteem with respect to mathematics is greatly enhanced.

That third and fourth graders can solve such equations at all is a tribute to the power available to students when an abstract concept is presented through a physical model that they can readily grasp. The power of such a physical model was noted as early as 1959 by Professor Bärbel Inhelder of Geneva:

> Advanced notions of mathematics are perfectly accessible to children of seven to ten years of age, *provided that they are divorced from their mathematical expression and studied through materials that the child can handle himself.*[1]

The Hands-On Equations Learning System presents just such a physical and intuitive model of the world of basic algebra! The students simply transform the given abstract equation into its physical counterpart, and then proceed to solve the given equation through the use of physical "legal moves."*

Level I of the program, which takes seven lessons, easily fits into the present elementary school curriculum at the third or fourth grade level. The time spent on *Hands-On Equations*, other than the time spent on critical thinking and analysis, is primarily time spent on mental arithmetic and basic facts - albeit a very enjoyable and meaningful way of doing so. Such time can easily replace other classroom time spent on mental arithmetic or on work with basic facts.

*The visual and kinesthetic approach of **Hands-On Equations** is valuable not just for elementary school students but for middle school students who have not yet had a formal course in algebra, for high school students who might otherwise not be exposed to algebra, and for college students and adults who did not have a successful school experience with algebra. The positive impact of the program upon student self-esteem and mathematical interest, however, is greatest when the program is presented to students in the elementary grades.

Since algebra is basic to the further study of mathematics and science, students who do not gain access to algebra will find the doors closed to many professional career opportunities. Traditionally, there has been a chasm in mathematics education, in that students have been expected to grasp the concepts, symbols, and methods of algebra at the abstract level in 8th or 9th grade. *An intuitive, concrete and pictorial foundation for algebra has been missing in mathematics education!*

The ***Hands-On Equations*** **powerful teaching methodology provides this entranceway to algebra.** *This methodology enables students from grades 3 to adult to meaningfully gain access to, and advance, in the world of basic algebra.*

Early success with algebraic concepts provides students with a tremendous sense of mathematical power and self-confidence, bolsters students' mathematical interests, and provides students with an intuitive and concrete foundation for later algebraic work.

Below follows a description of ***The Hands-On Equations Learning System,*** **Level I. This sequence of lessons, with students using the concrete materials of the program, has been found successful with students from ages 8 to 84.**

The Hands-On Equations® Learning System
Level I

Objective: By the end of the seventh lesson, elementary school students (third grade and up) will be able to physically set up and solve such equations as

$$2x + x + x + 2 = 2x + 10$$

and

$$2(x + 4) + x = x + 16.$$

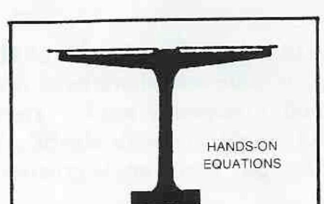
Materials Needed Per Student:

Eight blue pawns

Two red cubes, numbered 0-5

Two red cubes, numbered 5-10

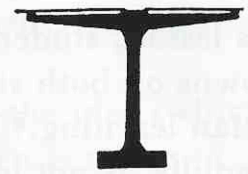
A laminated balance scale



<u>Materials Needed by the Teacher:</u>

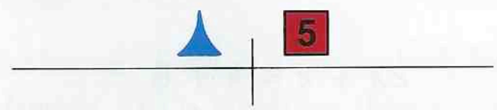
Pawns and cubes as above, but larger;

A stationary physical scale

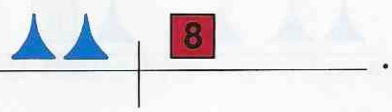


Lesson #1

In the first lesson, the teacher displays on the physical scale in front of the classroom, problems such as



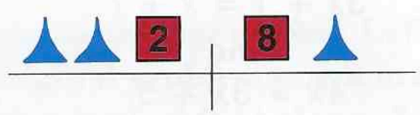
and



FOR HOME USE:
Please display these problems on the enclosed laminated balance scale.

Once students grasp the concept that both sides of the scale must have the same value for the scale to balance, they see that the pawn in the first problem is worth 5, and that in the second problem it is worth 4. Students can then be presented with other "physical equations" which they are to solve by trial and error methods.

Ex.



The students see that "1" does not work since both sides are not equal. "2" does not work, etc. "6" does work since the left side is now 14 and so is the right side. The students are informed that the pawn has a special name, "x," and that there is a special way of writing the answer:

$x = 6$, check: $14 \stackrel{\checkmark}{=} 14$.

The students are given Student Kits so that they can set up the worksheet problems at their desks. (On the student setup, it is helpful if the number-cubes are facing <u>upward</u>, i.e., facing the <u>ceiling</u>, so that the teacher can easily see if the students have the correct setup.)

Comments on Lesson #1

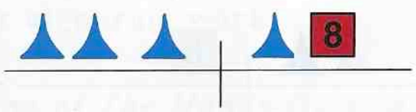
In this lesson, students begin learning about equations, variables, and unknowns on both sides of a setup—but they do so intuitively, through Piagetian learning.[2] Indeed, the word "variable," which can even scare some adults, is not used at all. Important algebraic concepts are nonetheless acquired in a very natural way as the students work with the materials.

Lesson #2

Students are reminded that the pawn has a special name, "x." Therefore, the problem

$$2x + x = x + 8$$

really calls for placing "two x's" and "one x" on the left side of the scale, and "an x" and an "8-cube" on the right side:



The students are given their Student Kits so that they can set up the problem at their desks. Then, they can solve by trial and error methods as in Lesson #1. The answer is:

$$x = 4, \text{ check: } 12 \stackrel{\checkmark}{=} 12.$$

Other examples which the teacher may use in this lesson include

$$3x + 1 = x + 7$$

and

$$4x = 3x + 5.$$

Comment on Lesson #2

It is a tremendous credit to the power of this system that young students can interpret and make sense of the above problems after only two lessons!

Lesson #3

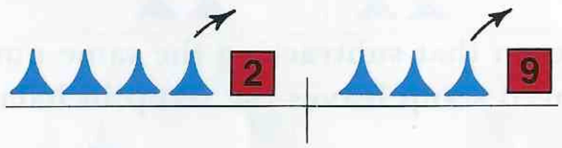
The teacher begins by posing to the class a problem such as

$$4x + 2 = 3x + 9.$$

The students set up this problem at their desks and attempt to solve it. Because this problem may stymie many students, it offers an excellent opportunity for the teacher to say:

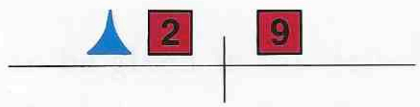
"Would you like to learn an easier way of getting the answer than by using trial and error?"

The teacher can now proceed to see if the students "buy" the idea (which the teacher now physically demonstrates), that if one pawn is removed simultaneously from each side of the balanced setup,



that the scale will still balance. *Such a move, which leaves a balanced system in balance, is called a "legal move."* (To confirm that the students do in fact understand this key concept, the teacher should then attempt to remove two blue pawns from the left side and one blue pawn from the right side. The students should see that this move is not legal.)

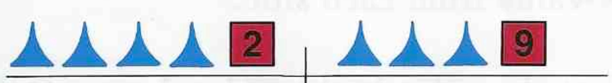
By carrying out the above legal move two more times, the setup now shows



from which the students can easily see that $x = 7$. The teacher can now say to the class:

> *"Let's see if our method has worked.* Let's physically set up the original problem one more time to see if $x = 7$ makes both sides balance."

A student can come up to the front of the room and, *after physically* resetting the original equation*, verify that if each pawn is worth 7, the system



balances since $30 \stackrel{\checkmark}{=} 30$. So the answer, $x = 7$, is correct.

*In **Hands-On Equations**, the check is always carried out in the original physical setup, **not** in the original abstract equation. The physical setup is the concrete meaning of the abstract equation.

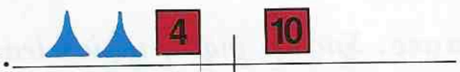
The teacher can now assign other similar problems for students to set up and solve at their seats, using legal moves if they wish:

Ex. $5x + 2 = 2x + 14$

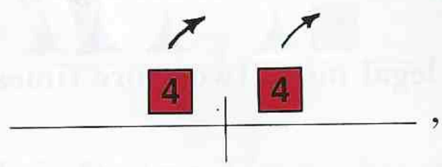
Ex. $2x + x + 4 = 4x + 1$

Lesson #4

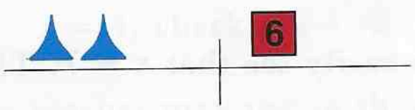
In this lesson, students learn that subtracting the same number-cube value from each side of a balanced setup leaves the setup in balance.

Ex. 

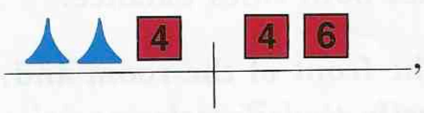
Given the above setup, students can subtract a 4-value from the cubes on each side,



thus leaving



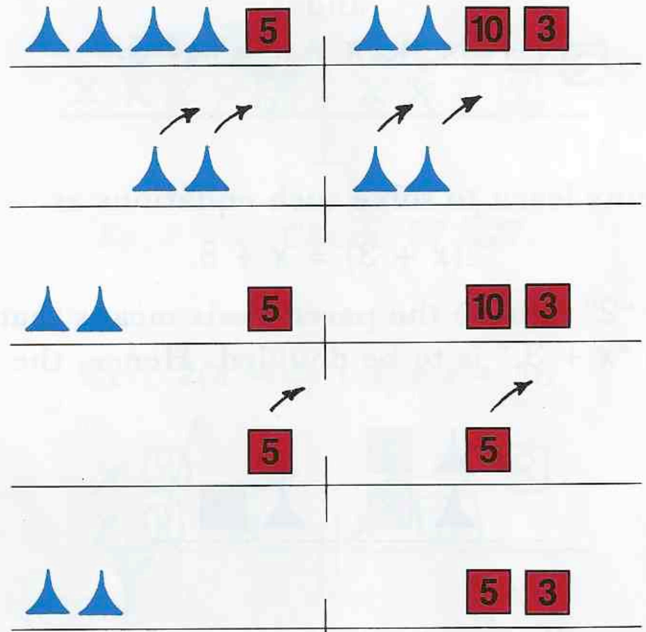
Sometimes this process is more clearly illustrated if the 10-cube is first replaced by a 4-cube and a 6-cube,



before removing the 4-value from each side.

So far, then, the students have learned two legal moves: that they may substract the same number of blue pawns, or X's, from each side of a balanced setup, or that they may subtract the same value from the cubes on each side. The following example enables the students to perform both of these legal moves. A possible solution sequence is shown.

Ex. $4x + 5 = 2x + 13$



So, $x = 4$. The check in the initial physical setup reveals that $21 \stackrel{\checkmark}{=} 21$.

Other problems which can be given in this lesson include

$$2x + x + x + 2 = 2x + 10$$

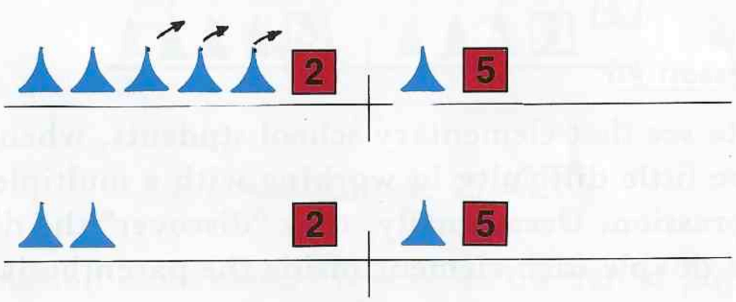
and

$$x + 3x + 3 = x + 18.$$

Lesson #5

In this lesson, students take away pawns as <u>part of the setup process:</u>

$$5x - 3x + 2 = x + 5$$



From this, the <u>original physical setup</u>, students can now proceed to use legal moves. After removing one blue pawn from each side, we see that $x = 3$. The check, in the above setup, shows that $8 \stackrel{\checkmark}{=} 8$.

Other examples which the teacher can assign in this lesson include

$$2x + x - x + 1 = x + 9$$

and

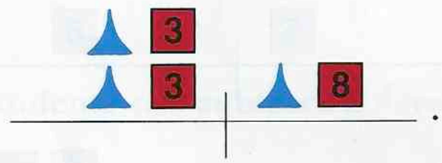
$$4 + 3x - 2x + x = x + 5.$$

Lesson #6

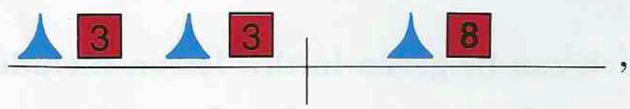
In this lesson, students learn to solve such equations as

$$2(x + 3) = x + 8.$$

They learn that the "2" outside the parenthesis means that what is inside the parenthesis, the "$x + 3$," is to be doubled. Hence, the student setup for this problem is



By having the students display the doubled portion in two rows on the mat, the teacher can easily check that the correct elements have been doubled. (In the teacher setup, the pawns and cubes are placed next to each other,



so that they are visible to the class.) By subtracting one x from each side, we see that $x = 2$; check $10 \stackrel{\checkmark}{=} 10$.

Other examples the teacher can give in this lesson include

$$2(2x + 1) = 18$$

and

$$2(x + 4) + x = x + 16.$$

Comments on Lesson #6

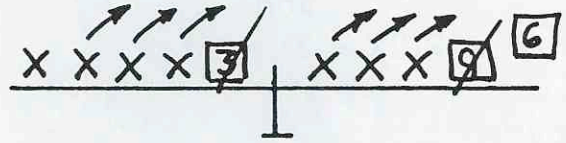
It is fascinating to see that elementary school students, when instructed in this manner, have little difficulty in working with a multiple of a parenthetical expression. Occasionally, they "discover" the distributive law on their own and double each element inside the parenthesis in sequence.

Lesson #7

In this lesson, students transfer their concrete, hands-on experience in solving algebraic linear equations to a pictorial system involving only pencil and paper. The technique is illustrated in the two examples below.

Ex. $4x + 3 = 3x + 9$

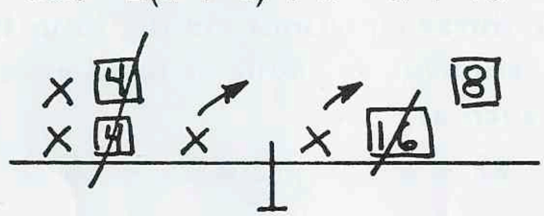
Solution:



So, $x = 6$. Check: $27 \stackrel{\checkmark}{=} 27$.

Ex. $2(x + 4) + x = x + 16$

Solution:



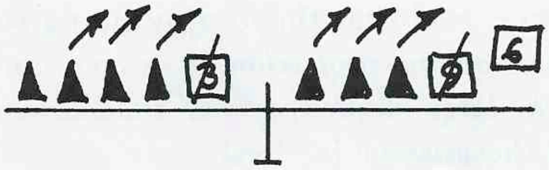
So, $x = 4$. Check: $20 \stackrel{\checkmark}{=} 20$.

Thus in this pictorial notation, the student draws a scale, and on it, places written "x's" (instead of pawns), places written boxed-numerals (instead of number-cubes), and crosses off or places arrows above anything that is to be taken away. Once the answer is obtained, the student goes back to the initial pictorial setup *(redrawn, for clarity)* in order to carry out the check.

<u>Note:</u> Some students prefer to use pictures of the pawns, rather than written "x's," to represent the physical pawns.

Ex. $4x + 3 = 3x + 9$

Solution:



So, $x = 6$. Check: $27 \stackrel{\checkmark}{=} 27$.

This pictorial notation more closely resembles the actual physical setup and is therefore easier for some younger students and some students with learning disabilities to understand. This notation is perfectly acceptable. (The picture of the blue pawn is shaded in, to distinguish it, in Level II, from the picture of the white pawn which is not shaded in.)

Other practice examples, to be solved pictorially, are

$$x + 2x + 14 = 5x + 2$$

and

$$2(2x + 3) = x + 9.$$

Comment on Lesson #7

The pictorial notation represents a transitional step from the concrete mode to the formal mode of solving algebraic linear equations.

Summary

By presenting algebraic linear equations via the intuitive, physical model of *Hands-On Equations*, students as young as third graders can experience success with equations such as

$$2x + x + x + 2 = 2x + 10$$

and

$$2(x + 4) + x = x + 16.$$

Once students learn to translate the algebraic linear equation into its physical counterpart and learn the concept of a "legal move," *they gain power in the world of algebra. The abstract has become tangible and understandable; the intimidating has become game-like.* Indeed, solving equations via *Hands-On Equations* becomes an experience that students enjoy and want to pursue.[3] This enjoyment of sophisticated-looking mathematics serves to enhance student self-esteem and to heighten student mathematical aspirations.

A Final Comment

Now that we know that elementary school students are capable of doing this kind of mathematics, and that they enjoy doing so, the question is upon us: *Will we in the teaching profession be as bold and daring as we need to be if we are to provide the large majority of our students with early access to algebra?* I believe that the answer is "Yes!"

Notes

1. This quote by Inhelder, a colleague of Piaget, was taken from a presentation made at the 1959 Woods Hole Conference held at Cape Cod. It is found in Jerome Bruner's *The Process of Education* (Vintage Books, 1960, p. 43).
2. "Piagetian learning" refers to "learning without being taught," as discussed by Seymour Papert in his book *Mindstorms* (Basic Books, Inc., 1980, p. 7). Some of our most powerful and lasting learnings, such as learning to speak, are carried out via Piagetian learning. ***Hands-On Equations*** *enables students to grasp key concepts of equations, variables, and algebraic notation through Piagetian learning.*
3. In his book, *The Quality School, Managing Students Without Coercion* (1990, Harper and Row, Publishers, New York), Dr. William Glasser asserts that once students have the experience of doing high quality work, they will find this experience highly satisfying and will therefore desire more similar experiences. I claim that for many students, **Hands-On Equations** is the breakthrough learning experience which reveals to them both the joy of learning and the tremendous learning capacity which they possess.